Engineering News

ALTEC

ALTEC 1515 S. Manchester Avenue, Anaheim, California

TECHNICAL LETTER NO. 212A

USEFUL EQUATIONS FOR THE SOUND CONTRACTOR

By

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The advent of very small, very powerful calculators, such as the H.P. 35, have made it convenient for sound contractors to directly use the original equation rather than a nomongraph developed from such equations. Greater accuracy to several more places is one reward of this approach. More importantly perhaps, is the familiarity with the equations themselves as many new and more usable combinations of them tend to suggest themselves.

Note: In this technical letter, $\ln \equiv \log_e$.

Basic Sound System Equations

The most used equations are those of ohms law:

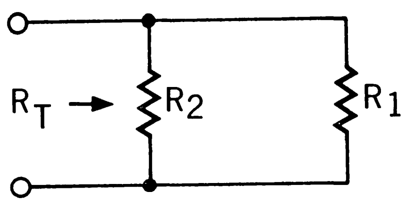
1. $E = \sqrt{WR}$, $E = \frac{W}{I}$, $E = IR$

2. $W = \frac{E^2}{R}$, $W = I^2R$, $W = EI$

3. $I = \frac{E}{R}$, $I = \frac{W}{E}$, $I = \sqrt{\frac{W}{R}}$

4. $R = \frac{E}{I}$, $R = \frac{E^2}{W}$, $R = \frac{W}{I^2}$

5. $R_T = \frac{R_1 \cdot R_2}{R_1 + R_2}$



6. $R_2 = \frac{R_1 \cdot R_T}{R_1 - R_T}$

7. $R_S = R_1 + R_2 + R_3 \quad \ldots + R_n$

8. $R_P = \cfrac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \quad \ldots + \frac{1}{R_n}}$

Litho in USA CP-572-2.5K

9. $$C_S = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \dots + \frac{1}{C_n}}$$

10. $$C_P = C_1 + C_2 + C_3 \dots + C_n$$

11. $$L_S = L_1 + L_2 + L_3 \dots + L_n$$

12. $$L_P = \frac{1}{\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} \dots + \frac{1}{L_n}}$$

Decibel Equations

Wherever a power ratio is expressed, it can be written as:

1. $$10 \log_{10} \left(\frac{P_1}{P_2} \right)$$

Wherever voltage, SPL etc., are expressed, they can be written as:

2. $$20 \log_{10} \left(\frac{E_1}{E_2} \right) \dots \left(\frac{SPL_1}{SPL_2} \right) \quad \text{etc. ,}$$

3. $$\text{Antilog}_{10} \left(\frac{dB}{10} \right) = 10^{\left(\frac{dB}{10} \right)}$$

4. $$\text{Antilog}_{10} \left(\frac{dB}{20} \right) = 10^{\left(\frac{dB}{20} \right)} = K$$

5. $$\text{Antilog}_{10} \left(\frac{dB}{10} \right) = e^{\left[\ln 10 \left(\frac{dB}{10} \right) \right]}$$

6. $$\text{Antilog}_{10} \left(\frac{dB}{20} \right) = e^{\left[\ln 10 \left(\frac{dB}{20} \right) \right]}$$

Time Delay Equations

With the introduction of digital time-delay devices and the likelihood of reducing costs in such devices, the following equations will be increasingly used.

To find time if distance is known:

$$T = D \left(\frac{1}{1.13} \right)$$

To find distance when time is known:

$$D = 1.13\ T$$

where D is in feet and T is in milliseconds.

Equations Dependent Upon Room Parameters

The equations all require that the calculator has found at least three of the following parameters:

V = internal volume of the space in ft^3

S = total boundary surface area in ft^2

RT_{60} = the reverberation time for a delay of 60 dB from a steady-state signal that is interrupted

$\bar{a}$ = average absorption coefficient

$$\bar{a} = \frac{s_1 a_1 + s_2 a_2 + s_3 a_3 \quad \ldots + s_n a_n}{S}$$

where: s = individual surface area in ft^2

a = absorption coefficient of individual surface per square foot

Knowledge of any three of these allows the rapid calculation of the missing parameter as the following four equations illustrate:

1. $$V = \frac{-S \ln(1 - \bar{a})\, RT_{60}}{0.049}$$

2. $$S = \frac{0.049}{-RT_{60} \ln(1 - \bar{a})} = \frac{R}{\bar{a}} - R$$

3. $$\bar{a} = 1 - e^{-\left(\frac{0.049V}{S \cdot RT_{60}}\right)} = \frac{R}{R + S}$$

4. $$RT_{60} = \frac{0.049V}{-S \ln(1 - \bar{a})}$$

Directivity Factor and Directivity Index

When the Directivity Factor (Q) and Directivity Index (DI) are introduced to the equations, room-loudspeaker interaction can be predicted by using the following equations:

1. For angles from 180° to 1° $$Q = \frac{180}{\arcsin\left(\sin \frac{\theta}{2}\right)\left(\sin \frac{\phi}{2}\right)}$$

2. $$DI = 10 \log_{10} Q$$

3. $$Q = \text{antilog}_{10}\left(\frac{DI}{10}\right)$$

4. $$DI = SPL_\theta - SPL_s$$

where: $$SPL_\theta = \left[10 \log_{10}\left(\frac{WQ10^{13}}{4\pi r^2}\right)\right] + 0.5$$

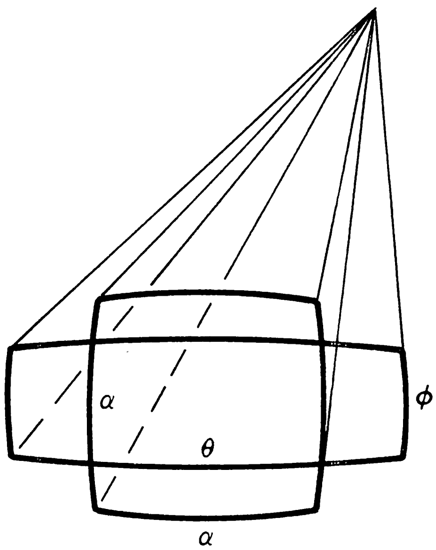
$$SPL_s = \left[10 \log_{10}\left(\frac{W10^{13}}{4\pi r^2}\right)\right] + 0.5$$

5. $$Q = \frac{180}{\arcsin\left(\frac{\sin^2 \alpha}{2}\right)}$$

6. $$\alpha = 2 \arcsin \sqrt{\sin \frac{180}{Q}}$$

7. $$\phi = 2 \arcsin \left(\frac{\frac{\sin^2 \alpha}{2}}{\sin \frac{\theta}{2}}\right)$$

8. $$\theta = 2 \arcsin \left(\frac{\frac{\sin^2 \alpha}{2}}{\sin \frac{\phi}{2}}\right)$$



Critical Distance Equations

One of the most useful room-sound system parameters is critical distance (D_c). This is the distance at which the direct sound source is equal to the reverberant sound generated by the enclosed space.

1. $$D_c = 0.141 \sqrt{QR}$$

2. $$4D_c = MAX\ D_2$$

3. $$\frac{dB_d}{dB_r} = 10 \log_{10} \left[\left(\frac{Q}{16\pi r^2}\right) (R) \right]$$

4. $$R = \frac{s\bar{a}}{1 - \bar{a}} = \frac{(D_c)^2}{0.019881\ Q}$$

5. $$s\bar{a} = R\ (1 - \bar{a})$$

6. $$\Delta D_x = 10 \log_{10} \left[\left(\frac{Q}{4\pi r^2}\right) + \left(\frac{4}{R}\right) \right]$$

7. $r = \sqrt{\left[\dfrac{Q}{\dfrac{\text{antilog}_{10}\left(\frac{\Delta D_x}{10}\right) - \left(\frac{4}{R}\right)}{4\pi}}\right]}$

where: dB = r desired

EXAMPLE: $\left[\text{PAG} - \text{NAG}\right] \pm \Delta D_x = \text{dB}$

8. given: $D_2 = 4D_c$

$$\text{MIN } Q = \frac{(0.25D2)^2}{0.019881\ R}$$

$$\text{MIN } R = \frac{(0.25D_2)^2}{0.019881\ Q}$$

TO FIND MAX RT_{60}:

(A) FIND MIN R

(B) FIND MIN $\bar{a}$, $\left(\text{MIN } \bar{a} = \dfrac{\text{MIN R}}{\text{MIN R} + S}\right)$

(C) CALCULATE MAX RT_{60}

9. $\text{NAG} = \Delta D_o - \Delta \text{EAD} + 10 \log_{10} \text{NOM} + 6$

10. $\text{PAG} = \Delta D_o + \Delta D_1 - \Delta D_s - \Delta D_2$

11. New D_s in dB = $\Delta D_s + (\text{PAG} - \text{NAG})$

12. New D_2 in dB = $\Delta D_2 + (\text{PAG} - \text{NAG})$

13. New D_1 in dB = $\Delta D_1 - (\text{PAG} - \text{NAG})$

14. New EAD in dB = $\Delta \text{EAD} - (\text{PAG} - \text{NAG})$

Finding the Efficiency of a Loudspeaker

The Q of an unknown loudspeaker can be found by measuring its D_c in a reverberant space. Once its Q is known, its sensitivity can be measured out of doors at 4' and then its efficiency can be calculated.

$$\%\ \text{Effic.} = \text{antilog}_{10} \left[\frac{\text{(L) Effic.} - (10 \log_{10} Q + 107.47)}{10}\right] \times 100$$

where: (L) Effic. = 4', 1 watt sensitivity

Summing Noise Levels

The Walsh-Healy Act has brought increased attention to the measurement of ambient noise levels. They are often measured by plotting 1/3-octave bands on a chart. When a summed reading was not taken on the C scale or linear scale of an SLM, the overall wideband SPL can be calculated with the following equations:

$$CNL = 10 \log_{10}\left[\left(\text{antilog}_{10}\frac{dB}{10}\right)_1 + \left(\text{antilog}_{10}\frac{dB}{10}\right)_2 \quad \ldots + \left(\text{antilog}_{10}\frac{dB}{10}\right)_n\right]$$

where: CNL = combined bands noise level

Calculating THD

When a wave analyzer has been used to obtain the number of dB below the fundamental of each harmonic, THD can be calculated with the following equation:

$$THD = \sqrt{\left(\frac{10,000}{\text{antilog}_{10}\frac{dB}{10}}\right)_1 + \left(\frac{10,000}{\text{antilog}_{10}\frac{dB}{10}}\right)_2 \quad \ldots + \left(\frac{10,000}{\text{antilog}_{10}\frac{dB}{10}}\right)_n}$$

Vector Calculation

The larger programmable calculators offer rectangular-to-polar and polar-to-rectangular conversion. H.P. 35 does not offer this conversion, but such calculations are relatively simple:

Rectangular-to-Polar Conversion

1. Vector length $= \sqrt{x^2 + y^2}$

2. Angle $= \arctan \frac{y}{x}$

$= \arctan \frac{-y}{x}$

$= \arctan \frac{y}{-x} + 180°$

$= \arctan \frac{-y}{-x} - 180°$

Polar-to-Rectangular Conversion

1. Sin angle times vector = y
   Angle 0° to + 180° = + y
   Angle 0° to - 180° = - y

2. Cos angle times vector = x
   Angle 0° to + 90° = + x
   Angle 0° to - 90° = + x
   Angle + 90° to + 180° = - x
   Angle - 90° to - 180° = - x

Calculating Hyperbolic Functions

While hyperbolic functions are not used frequently in sound contracting work, they are valuable tools for loudspeaker designers, communication-system engineers, etc. The H.P. 35 can be used to quickly find any hyperbolic function by using the following equations:

1. $\sinh x = \dfrac{e^x - e^{-x}}{2}$

2. $\text{arc} \sinh x = \ln \left[x + \left(x^2 + 1 \right)^{1/2} \right]$

3. $\cosh x = \dfrac{e^x + e^{-x}}{2}$

4. $\text{arc} \cosh x = \ln \left[x + \left(x^2 - 1 \right)^{1/2} \right]$

5. $\text{Tanh } x = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$

6. $\text{arc} \tanh x = \dfrac{1}{2} \left[\ln \left(\dfrac{1+x}{1-x} \right) \right]$

Equations for Articulation Losses

1. Maximum D_2 for AL_{cons} of 15% $= \sqrt{\dfrac{15\,VQ}{641.81\,(RT_{60})^2}}$

2. Maximum RT_{60} for AL_{cons} of 15% $= \sqrt{\dfrac{15VQ}{641.81\,(D_2)^2}}$

3. Maximum V for AL_{cons} of 15% $= \dfrac{641.81\,(D_2)^2\,(RT_{60})^2}{15Q}$

4. Minimum Q for AL_{cons} of 15% $= \dfrac{641.81\,(D2)^2\,(RT_{60})^2}{15V}$

5. AL_{cons} in percentage $= \dfrac{641.81\,(D2)^2\,(RT_{60})^2}{VQ}$

SUMMARY

These useful equations constitute a minimum beginning for workers in the sound contracting business. It is hoped to expand this collection year by year into catalogues that will serve as a ready and useful compilation of equations for the busy sound system engineer. Suggestions for additions in both catalogues and equations are solicited and welcome here in Anaheim.

APPENDIX

MATHEMATICAL SYMBOLS

X or ·	Multiplied by
÷ or :	Divided by
+	Positive. Plus. Add
-	Negative. Minus. Subtract
±	Positive or negative. Plus or minus
∓	Negative or positive. Minus or plus
= or ::	Equals
≡	Identity
≅	Is approximately equal to
≠	Does not equal
>	Is greater than
≫	Is much greater than
<	Is less than
≪	Is much less than
≧	Greater than or equal to
≦	Less than or equal to
∴	Therefore
∠	Angle
Δ	Increment or decrement
⊥	Perpendicular to
‖	Parallel to
$\lvert n \rvert$	Absolute value of

MATHEMATICAL CONSTANTS

π	= 3.1415962654		$\sqrt{\pi}$	= 1.7722453851
2π	= 6.283185307		$\sqrt{2}$	= 1.414213562
$(2\pi)^2$	= 39.47841760		$\sqrt{3}$	= 1.732050808
4π	= 12.56637061		$\frac{1}{\sqrt{2}}$	= 0.707106781
π^2	= 9.869604401		$\frac{1}{\sqrt{3}}$	= 0.577350269
$\frac{\pi}{2}$	= 1.570796327		$\log \pi$	= 0.497149873
$\frac{1}{\pi}$	= 0.318309886		$\log \frac{\pi}{2}$	= 0.196119877
$\frac{1}{2\pi}$	= 0.159154943		$\log \pi^2$	= 0.994299745
$\frac{1}{\pi^2}$	= 0.101321184		$\log \sqrt{\pi}$	= 0.248574936
$\frac{1}{\sqrt{\pi}}$	= 0.564189584		e	= 2.718281828

ALGEBRA

Exponents and Radicals

$a^x \times a^y = a^{(x+y)}$	$\frac{a^x}{a^y} = a^{(x-y)}$
$(ab)^x = a^x b^x$	$\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$
$\sqrt[x]{\frac{a}{b}} = \frac{\sqrt[x]{a}}{\sqrt[x]{b}}$	$a^{-x} = \frac{1}{a^x}$
$(a^x)^y = a^{xy}$	$\sqrt[x]{\sqrt[y]{a}} = \sqrt[xy]{a}$
$\sqrt[x]{ab} = \sqrt[x]{a}\,\sqrt[x]{b}$	$a^{\frac{x}{y}} = \sqrt[y]{a^x}$
$a^{\frac{1}{x}} = \sqrt[x]{a}$	$a^0 = 1$

Solution of a Quadratic

Quadratic equations in the form

$$ax^2 + bx + c = 0$$

may be solved by the following:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Transposition of Terms

If $A = \frac{B}{C}$, then $B = AC$, $C = \frac{B}{A}$

If $\frac{A}{B} = \frac{C}{D}$, then $A = \frac{BC}{D}$, $B = \frac{AD}{C}$,

$C = \frac{AD}{B}$, $D = \frac{BC}{A}$

If $A = \frac{1}{D\sqrt{BC}}$, then $A^2 = \frac{1}{D^2BC}$,

$B = \frac{1}{D^2A^2C}$, $C = \frac{1}{D^2A^2B}$, $D = \frac{1}{A\sqrt{BC}}$

If $A = \sqrt{B^2 + C^2}$, then $A^2 = B^2 + C^2$,

$B = \sqrt{A^2 - C^2}$, $C = \sqrt{A^2 - B^2}$